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Investigation of Antiplane Shear Behavior of Two Collinear Permeable Cracks in a Piezoelectric Material by Using the Nonlocal Theory

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1 Introduction

In the theoretical studies of crack problems, several different electric boundary conditions at the crack surfaces in piezoelectric materials have been proposed by numerous researchers ([1–4]). However, these solutions contain stress and electric displacement singularity. This is not reasonable according to the physical nature. To overcome the stress singularity in the classical elastic theory, Eringen [5] used the nonlocal theory to study the state of stress near the tip of a sharp line crack in an elastic plate subjected to antiplane shear. The solution did not contain any stress singularity. Recently, the same problems have been resolved in Zhou's papers ([6]) by using the Schmidt method.

In this paper, the behavior of two collinear symmetric cracks subjected to the antiplane shear loading in the piezoelectric materials is investigated by using the Schmidt method and the nonlocal theory for permeable crack surface conditions. The traditional concept of linear elastic fracture mechanics and the nonlocal theory are extended to include the piezoelectric effects. As expected, the solution in this paper does not contain the stress and electric displacement singularity at the crack tip.

2 Basic Equations of Nonlocal Piezoelectric Materials

As discussed in [7], for the antiplane shear problem, the basic equations of linear, nonlocal piezoelectric materials can be written as follows:

$$\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}=0 \tag{1}$$

$$\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}=0 \tag{2}$$

$$\tau_{kz}(X)=\int_V \alpha(|X'-X|)[c_{44}w_{,k}(X')+e_{15}\phi_{,k}(X')]dV(X'),\quad (k=x,y) \tag{3}$$

$$D_k(X)=\int_V \alpha(|X'-X|)[e_{15}w_{,k}(X')-\varepsilon_{11}\phi_{,k}(X')]dV(X'),\quad (k=x,y) \tag{4}$$

where the only difference from classical elastic theory is that in the stress and the electric displacement constitutive Eqs. (3)–(4), the stress $\tau_{zk}(X)$ and the electric displacement $D_k(X)$ at a point X depends on $w_{,k}(X)$ and $\phi_{,k}(X)$, at all points of the body. w and ϕ are the mechanical displacement and electric potential. $c_{44},e_{15},\varepsilon_{11}$ are the shear modulus, piezoelectric coefficient, and dielectric parameter, respectively. $\alpha(|X'-X|)$ is the influence function. As discussed in the papers ([5,6]), $\alpha(|X'-X|)$ can be assumed as follows:

$$\alpha(|X'-X|)=\frac{1}{\pi}(\beta/a)^2\exp[-(\beta/a)^2(X'-X)(X'-X)] \tag{5}$$

where β is a constant and a is the lattice parameter.

3 The Crack Model

Consider an infinite piezoelectric plane containing two collinear symmetric permeable cracks of length 1-b along the x-axis. $2b$ is the distance between two cracks. The boundary conditions of the present problem are

$$\tau_{yz}^{(1)}(x,0^+)=\tau_{yz}^{(2)}(x,0^-)=-\tau_0,\quad b\leqslant|x|\leqslant 1 \tag{6}$$

$$D_y^{(1)}(x,0^+)=D_y^{(2)}(x,0^-),\quad \phi^{(1)}(x,0^+)=\phi^{(2)}(x,0^-),\quad |x|\leqslant\infty \tag{7}$$

$$w^{(1)}(x,0^+)=w^{(2)}(x,0^-)=0,\quad 0<|x|<b,1<|x| \tag{8}$$

$$w^{(k)}(x,y)=\phi^{(k)}(x,y)=0,\quad \text{for } (x^2+y^2)^{1/2}\to\infty,\quad (k=1,2). \tag{9}$$

Note that all quantities with superscript $k(k=1,2)$ refer to the upper half-plane and the lower half-plane.

As discussed in [7], the general solutions of Eqs. (1)–(2) satisfying (9) are, respectively,

$$w^{(1)}(x,y)=\frac{2}{\pi}\int_0^\infty A_1(s)e^{-sy}\cos(xs)ds,$$

$$\phi^{(1)}(x,y)-\frac{e_{15}}{\varepsilon_{11}}w^{(1)}(x,y)=\frac{2}{\pi}\int_0^\infty B_1(s)e^{-sy}\cos(xs)ds \tag{10}$$

$$w^{(2)}(x,y)=\frac{2}{\pi}\int_0^\infty A_2(s)e^{sy}\cos(xs)ds,$$

$$\phi^{(2)}(x,y)-\frac{e_{15}}{\varepsilon_{11}}w^{(2)}(x,y)=\frac{2}{\pi}\int_0^\infty B_2(s)e^{sy}\cos(xs)ds \tag{11}$$

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where $A_1(s)$, $B_1(s)$, $A_2(s)$, $B_2(s)$ are to be determined from the boundary conditions.

For solving the problem, the gap functions of the crack surface displacements and the electric potentials are defined as follows:

$$f_w(x)=w^{(1)}(x,0^+)-w^{(2)}(x,0^-) \tag{12}$$

$$f_\phi(x)=\phi^{(1)}(x,0^+)-\phi^{(2)}(x,0^-). \tag{13}$$

Substituting Eqs. (10)–(11) into Eqs. (3)–(4), (12)–(13), applying the Fourier transform and the boundary conditions (6)–(8), it can be obtained as

$$\frac{1}{\pi}\int_0^\infty s\bar{f}_w(s)\mathrm{erfc}(\varepsilon s)\cos(sx)ds=\frac{\tau_0}{c_{44}},\quad b\leqslant|x|\leqslant 1 \tag{14}$$

$$\frac{1}{\pi}\int_0^\infty \bar{f}_w(s)\cos(sx)ds=0,\quad 0<|x|<b,\quad 1<|x|<\infty \tag{15}$$

and $\bar{f}_\phi(s)=0$, $f_\phi(x)=0$, for all s and x. $\varepsilon=a/2\beta$, $\mathrm{erfc}(z)=1-\Phi(z)$, $\Phi(z)=2/\sqrt{\pi}\int_0^z\exp(-t^2)dt$.

4 Solution of the Triple Integral Equations

As discussed in [6], the Schmidt method ([8]) can be used to solve the triple-integral Eqs. (14)–(15). The gap functions of the crack surface displacement can be represented by the following series:

$$f_w(x)=\sum_{n=0}^\infty a_n P_n^{(1/2,1/2)}\left(\frac{x-\frac{1+b}{2}}{\frac{1-b}{2}}\right)\times\left(1-\frac{\left(x-\frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{1/2},\quad \text{for } b\leqslant x\leqslant 1,\ y=0 \tag{16}$$

$$f_w(x)=0,\quad \text{for } 0<x<b,\ 1<x,\ y=0 \tag{17}$$

where a_n is unknown coefficients to be determined and $P_n^{(1/2,1/2)}\times(x)$ is a Jacobi polynomial. The Fourier transformation of Eq. (16) is

$$\bar{f}_w(s)=\sum_{n=0}^\infty a_n Q_n G_n(s)\frac{1}{s}J_{n+1}\left(s\frac{1-b}{2}\right) \tag{18}$$

$$Q_n=2\sqrt{\pi}\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!},$$

$$G_n(s)=\begin{cases}(-1)^{n/2}\cos\left(s\frac{1+b}{2}\right), & n=0,2,4,6,\ldots\\ (-1)^{(n+1)/2}\sin\left(s\frac{1+b}{2}\right), & n=1,3,5,7,\ldots\end{cases} \tag{19}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively. By substituting Eq. (18) into Eqs. (14)–(15), respectively, Eq. (15) can be automatically satisfied. Then the remaining Eq. (14) reduces to the form

$$\sum_{n=0}^\infty a_n Q_n\int_0^\infty \mathrm{erfc}(\varepsilon s)G_n(s)J_{n+1}\left(s\frac{1-b}{2}\right)\cos(sx)ds=\frac{\pi}{c_{44}}\tau_0. \tag{20}$$

Equations (20) can now be solved for the coefficients a_n by the Schmidt method ([8]).

5 Numerical Calculations and Discussion

τ_{yz} and D_y along the crack line can be expressed as

$$\tau_{yz}=\tau_{yz}^{(1)}(x,0)=-\frac{c_{44}}{\pi}\sum_{n=0}^\infty a_n Q_n\int_0^\infty \mathrm{erfc}(\varepsilon s)G_n(s)J_{n+1}\left(s\frac{1-b}{2}\right)\cos(xs)ds \tag{21}$$

$$D_y=D_y^{(1)}(x,0)=-\frac{e_{15}}{\pi}\sum_{n=0}^\infty a_n Q_n\int_0^\infty \mathrm{erfc}(\varepsilon s)G_n(s)J_{n+1}\left(s\frac{1-b}{2}\right)\cos(xs)ds=\frac{e_{15}}{c_{44}}\tau_{yz}^{(1)}(x,0). \tag{22}$$

So long as $\varepsilon\neq 0$, the semi-infinite integration and the series in Eqs. (20) is convergent for any variable x. Equations (21) and (22) give finite stress and electric displacement all along $y=0$, so there are no stress and electric displacement singularity at the crack tips. The results are plotted in Figs. 1 and 2. From the results, the dimensionless stress field is found to be independent of the material parameters. They just depend on the length of the crack and the lattice parameter. However, the electric displacement field is found to depend on the stress loads, the shear modulus, the length of the crack, the lattice parameter and piezoelectric coefficient except the dielectric parameter ε_{11}. Contrary to the impermeable crack surface condition solution, it is found that the electric displacement for the permeable crack surface conditions is much smaller than the results for the impermeable crack surface conditions.

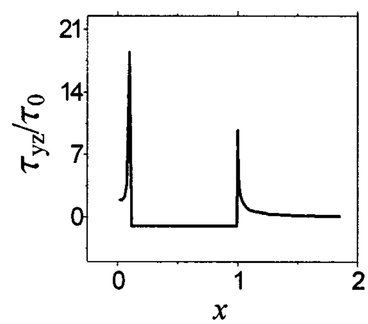

Fig. 1 The stress along the crack line versus *x* for *b*=0.1, *a*/2*β*=0.0005 (PZT-5H)

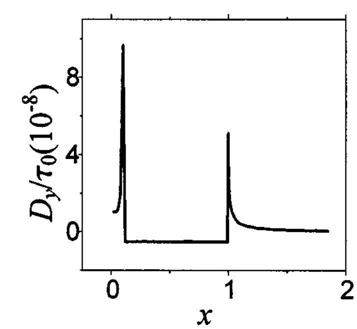

Fig. 2 The electric displacement along the crack line versus *x* for *b*=0.1, *a*/2*β*=0.0005 (PZT-5H)

References

[1] Deeg, W. E. F., 1980, The Analysis of Dislocation, Crack and Inclusion Problems in Piezoelectric Solids, Ph.D. thesis, Stanford University, Stanford, CA.

[2] Pak, Y. E., 1992, ''Linear Electro-Elastic Fracture Mechanics of Piezoelectric Materials,'' Int. J. Fract., **54**, pp. 79–100.

[3] Suo, Z., Kuo, C.-M., Barnett, D. M., and Willis, J. R., 1992, ''Fracture Mechanics for Piezoelectric Ceramics,'' J. Mech. Phys. Solids, **40**, pp. 739–765.

[4] Zhang, T. Y., and Hack, J. E., 1992, ''Mode-III Cracks in Piezoelectric Materials,'' J. Appl. Phys., **71**, pp. 5865–5870.

[5] Eringen, A. C., 1979, ''Linear Crack Subject to Antiplane Shear,'' Eng. Fract. Mech., **12**, pp. 211–219.

[6] Zhou, Z. G., Han, J. C., and Du, S. Y., 1999, ''Investigation of a Crack Subjected to Antiplane Shear by Using the Nonlocal Theory,'' Int. J. Solids Struct., **36**, No. 26, pp. 3891–3901.

[7] Zhou, Z. G., and Wang, B., 2001, ''Investigation of Antiplane Shear Behavior of Two Collinear Cracks in the Piezoelectric Materials by Using the Nonlocal Theory,'' Int. J. Solids Struct., submitted for publication.

[8] Morse, P. M., and Feshbach, H., 1958, *Methods of Theoretical Physics*, Vol. 1, McGraw-Hill, New York.